Approximate Analytical Evaluation of the Continuous Spectrum in a Substrate-Superstrate Dielectric Waveguide

Paolo Baccarelli*, Paolo Burghignoli*, Fabrizio Frezza*,
Alessandro Galli*, Giampiero Lovat*, and David R. Jackson**

* "La Sapienza" University of Rome, Electronic Engineering Department, Rome, Italy.
** University of Houston, Houston, TX, U.S.A.

Abstract — **In this work, an original closed-form approximate evaluation is performed for the continuous-spectrum field excited by an electric-current line source in a substrate-superstrate configuration, optimized for leaky-wave radiation. The validity of these results is shown for the near and the far field at different frequencies, including the frequency range in which the leaky wave is physical and the entire transition region through the spectral gap. This new closed-form result shows explicitly the nature of the continuous-spectrum field in the transition region, and provides insight into the nature of the fields on more complicated structures such as microwave integrated circuits.**

I. Introduction and Background

The two-layer planar dielectric waveguide shown in Fig. 1(a) (substrate-superstrate configuration) was proposed as an antenna geometry capable of producing narrow radiated beams, when the dimensions are chosen properly and the superstrate permittivity is high [1]. Its behavior has subsequently been interpreted as due to the excitation of leaky waves supported by the structure [2]. As is well known, under suitable conditions these solutions can represent in a highly-convergent manner the continuous-spectrum field in open waveguides [3,4].

The part of the continuous spectrum which is not represented by the leaky-wave field has recently been studied in [5], where it was termed the *residual wave*. The residual-wave contribution to the field becomes non-negligible in the neighborhood of the transition region, which typically exists between the physical leaky-wave region and the region where the bound-wave exists [6].

In this work we present an approximate closed-form expression for the residual-wave field on the air-dielectric interface, when the structure is excited by an infinite electric-current line source. This leads to a closed-form expression for the continuous-spectrum field on the interface. This representation is in terms of the contribution of two poles of the Green function, each weighted by an appropriate *transition function*.

This paper is organized as follows. In Section II the analytical derivation of the approximate closed-form expression for the continuous spectrum is summarized. In Section III numerical results are presented for a typical optimized structure, for both the aperture field and the radiated far field, and a comparison is presented with the results obtained through the use of an existing empirical transition function [7]. Finally, in Section IV conclusions are presented.

II. Analysis

The reference structure is a double-layer dielectric configuration on a ground plane, infinite along the z direction, excited by an infinite electric line source directed along the y axis, as shown in Fig. 1(a). With reference to the symbols defined in Fig. 1(a), we assume that the physical parameters satisfy the following relations [1,2,7]:

$$\begin{aligned} k_0 b\sqrt{\varepsilon_{r1} - \sin^2\theta_p} &= \pi \\ k_0 t\sqrt{\varepsilon_{r2} - \sin^2\theta_p} &= \pi/2 \\ x_0 &= b/2 \end{aligned} \qquad (1)$$

which optimize radiation at angle θ_p due to the first higher-order TE mode (TE_2) in a leaky regime.

The y-polarized electric field at the air-dielectric interface can be expressed by an inverse Fourier transform as:

$$E_y(h,z) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \tilde{G}_{yy}(h,k_z)e^{-jk_z z}dk_z\,, \qquad (2)$$

where the spectral Green function $\tilde{G}_{yy}$ is known in a simple closed form [4], and the integral is performed along the real axis, assuming infinitesimal losses. By

0-7803-7239-5/02/$10.00 © 2002 IEEE

deforming the integration path around the branch cut which defines the proper and improper Riemann sheets (see Fig. 1(b)), the total field can be expressed as the residue contribution of the captured proper real poles, which constitutes the bound-wave field (in our case, from the TE_1 mode) and the contribution of the integral around the branch cut, which constitutes the continuous-spectrum field. By further deforming the integration path to the steepest-descent path (SDP) (the vertical path C_{k0} that goes around the k_0 branch point in Fig. 1(b)), the continuous spectrum can be expressed as the sum of the residue contribution of the captured (physical) leaky poles (in our case, the TE_2 mode, neglecting the higher-order leaky modes) and the contribution of the integral around C_{k0}, which constitutes the residual-wave field [5] (called the space-wave field in [7]). The residual-wave field is therefore expressed as

$$\begin{aligned} E_y^{res}(h,z) &= \frac{1}{2\pi}\int_{C_{k0}} \widetilde{G}_{yy}(h,k_z)e^{-jk_z z}dk_z \\ &= -j\frac{e^{-jk_0 z}}{2\pi}\int_0^{+\infty} F(s)\,e^{-sz}ds \end{aligned} \quad . \tag{3}$$

In Eq. (3), the change of variable $k_z = k_0 - j\,s$ has been performed, and the resulting integrand $F(s)$ is the difference of the original integrand calculated on the two parts, proper and improper, of the C_{k0} path.

To obtain an approximate analytical expression for the residual-wave field, we make the following basic assumptions: first, we let s tend to zero, in order to obtain an asymptotic approximation for large z via Watson's lemma; second, we let the frequency f tend to the cutoff frequency f_{co} of the TE_2 mode, since we are interested in studying the neighborhood of the transition region between the leaky-wave and bound-wave regimes of this mode. Based on these assumptions, the following approximate form for $F(s)$ can be obtained:

$$F(s) \cong A\left(\frac{1}{s-s_1}-\frac{1}{s-s_2}\right)\sqrt{s}\ , \tag{4}$$

where A is a coefficient that depends on the involved physical parameters, while s_1 and s_2 are the locations of the poles of the layered Green's function in the two-sheeted s-plane, corresponding to the poles k_{zLW1} and k_{zLW2} shown in Fig. 1(b), respectively. From the behavior of $F(s)$ in the neighborhood of $s = 0$, it can be observed that the asymptotic decay of the continuous spectrum for high z is algebraic [5], and goes as $z^{-3/2}$ except at cutoff, where it goes as $z^{-1/2}$ [7].

The following closed-form expression for the approximate residual-wave field (ARW) can then be derived from Eqs. (3) and (4):

$$\begin{aligned} E_y^{res}(h,z) \cong \frac{A}{2}e^{-jk_0 z}\,\cdot \\ \Big\{\sqrt{s_1}\,e^{-s_1 z}\big[Sgn[\Im m[s_1]] + Erf(j\sqrt{s_1 z})\big] \\ -\sqrt{s_2}\,e^{-s_2 z}\big[Sgn[\Im m[s_2]] + Erf(j\sqrt{s_2 z})\big]\Big\} \end{aligned} \tag{5}$$

The approximate continuous-spectrum field (ACS) can be obtained by summing the leaky-wave field of the k_{zLW1} pole (when captured) and the approximate residual-wave field of Eq. (5). The result can be cast in the following form:

$$E_y^{cs}(h,z) \cong E_{LW1}(h,z)T_{LW1}(z) + E_{LW2}(h,z)T_{LW2}(z)\ , \tag{6}$$

where we have introduced the leaky-wave fields E_{LW1} due to the residue from the k_{zLW1} pole,

$$E_{LW1}(h,z) \cong A\sqrt{s_1}\,e^{-jk_{zLW1}z}\ , \tag{7}$$

with a similar expression for E_{LW2}, and the weighting (transition) functions T_{LW1} and T_{LW2} defined as:

$$T_{LWi}(z) = \frac{1}{2}+\frac{1}{2}Erf(j\sqrt{s_i z})\ , \tag{8}$$

where $i = 1,2$ and a suitable determination of the square-root function must be used, related to the improper determination of the transverse wavenumber. It should be noted that, in order to obtain an accurate representation of the continuous spectrum, the contributions of both poles k_{zLW1} and k_{zLW2} are necessary; these poles can be a complex-conjugate pair, or two improper real poles, depending on frequency. The first pole, which is captured when physical, has its residue contribution weighted by the standard transition function T_{LW1} (see, e.g., [4]). The second pole is never captured; however its residue-like contribution is also taken into account in Eq. (6), weighted by the *same* transition function T_{LW2}. The consideration of the nonphysical pole k_{zLW2} is necessary due to the fact that it is close to the k_0 branch point, and therefore influences the asymptotic evaluation [4,8,9].

A closed-form expression for the far field of the residual wave (not reported here for brevity) can also be obtained, by Fourier transforming the aperture field of Eq. (5).

III. Numerical Results

To demonstrate the validity of the proposed formulation, different numerical results will be presented, both in the near and in the far field, for the structure of Fig. 1(a), optimized according to Eqs. (1) for radiation at endfire ($\theta_p = \pi/2$) at a frequency $f = 19.5$ GHz.

In Fig. 2(a), a comparison is shown between the exact continuous spectrum (ECS), calculated numerically by subtracting the bound-wave TE_1 field from the total field, and the approximate continuous spectrum (ACS), which in this case is represented by the residual wave alone, evaluated according to Eq. (6), at the cutoff frequency of the TE_2 mode ($f = 19.5$ GHz). The relevant location of the pole singularities in the SDP plane is shown in the insert: it can be noted that one pole (k_{zLW1}) is at the k_0 branch point, while the other pole (k_{zLW2}) is an improper real pole. The agreement is seen to be excellent. In Fig. 2(b) the far field is shown for the same case. It is observed that the agreement between the total field (TF) and the ACS is excellent, especially near endfire.

In Fig. 3(a), a comparison of the field on the interface (radiating aperture) is shown at a lower frequency ($f = 19.4844$ GHz), for which the leaky-wave pole k_{zLW1} is very close to the SDP but is not yet captured, as can be seen in the insert. Here again the agreement between the ECS and ACS is excellent. In this case, since the poles k_{zLW1} and k_{zLW2} are complex, the empirical power-based transition function given in [7] can also be used. The resulting power-based leaky-wave field (PBLW) is also shown in Fig. 3(a). Although the PBLW field does not agree at all with the ECS on the interface (as expected, since this field by itself does not approximate the continuous spectrum), the far field from the PBLW aperture field matches very well with the exact far field, as shown in Fig. 3(b). This interesting observation was noted in [7].

In Fig. 4(a), a comparison of the aperture field is shown at $f = 19.4781$ GHz, for which the leaky-wave pole k_{zLW1} is captured, but is still very close to the SDP (see insert). In this case, the continuous spectrum field is the sum of the leaky-wave field (LW) and the residual-wave field, approximated by the ARW of Eq. (5). It can be seen that neither the LW nor the ARW field by itself accurately represents the exact continuous spectrum, while their sum, the ACS field, does. Also, once again, while the PBLW field does not accurately approximate the aperture field, it accurately reproduces the far field (as does the field produced by the ACS interface field).

At lower frequencies, the leaky-wave field accurately represents the continuous spectrum while the residual-wave field is negligible, though it is still well represented by Eq. (5) (results have to be omitted here).

IV. Conclusion

An original closed-form approximate expression for the continuous-spectrum aperture field has been formulated for a two-layer dielectric leaky-mode waveguide excited by an electric line source. The continuous-spectrum field is represented in the form of a weighted sum of two leaky-pole contributions: one corresponds to the leaky mode that forms the physical radiating beam, and the other corresponds to the nonphysical complex-conjugate pole. Numerical results show that the closed-form approximation is accurate for the field on the aperture, and for the far field that comes from it, in the *entire transition region* of the relevant leaky mode. This new approximate closed-form expression for the continuous spectrum provides the opportunity to analytically explore the nature of the fields produced by a practical source in the transition region, as the leaky-mode pole moves through the spectral-gap region. Although formulated for a simple two-layer dielectric structure, many of the conclusions should remain valid for sources on practical microwave integrated-circuit structures that support leaky modes.

References

[1] D. R. Jackson and N. G. Alexópoulos, "Gain enhancement methods for printed circuit antennas," *IEEE Trans. Antennas Propagat.*, Vol. 33, 9, pp. 976-987, Sept. 1985.

[2] D. R. Jackson and A. A. Oliner, "A leaky-wave analysis of the high-gain printed antenna configuration," *IEEE Trans. Antennas Propagat.*, Vol. 36, 7, pp. 905-910, July 1988.

[3] T. Tamir and A. A. Oliner, "Guided complex waves. Part I: Fields at an interface. Part II: Relation to radiation patterns," *Proc. IEE*, Vol. 110, pp. 310-334, Feb. 1963.

[4] L. Felsen and N. Marcuvitz, *Radiation and scattering of waves*. Prentice-Hall, Englewood Cliffs, NJ, 1973.

[5] D. R. Jackson, F. Mesa, M. J. Freire, D. P. Nyquist, and C. Di Nallo, "An excitation theory for bound modes, leaky modes, and residual-wave currents on stripline structures," *Radio Sci.*, Vol. 35, 2, pp. 495-510, March-April 2000.

[6] P. Lampariello, F. Frezza, and A. A. Oliner, "The transition region between bound-wave and leaky-wave ranges for a partially dielectric-loaded open guiding structure," *IEEE Trans. Microwave Theory Tech.*, Vol. 38, pp. 1831-1836, Dec. 1990.

[7] H. Ostner, J. Detlefsen, and D. R. Jackson, "Radiation from one-dimensional dielectric leaky-wave antennas," *IEEE Trans. Antennas Propagat.*, Vol. 43, 4, pp. 331-339, Apr. 1995.

[8] M. Marin, S. Barkeshli, and P. H. Pathak, "Efficient analysis of planar geometries using a closed-form asymptotic representation of the grounded dielectric slab Green's function," *IEEE Trans. Microwave Theory Tech.*, Vol. 37, 4, pp. 669-679, Apr. 1989.

[9] M. Marin and P. H. Pathak, "An asymptotic closed-form representation for the grounded double-layer surface Green's function," *IEEE Trans. Antennas Propagat.*, Vol. 41, 11, pp. 1357-1366, Nov. 1992.

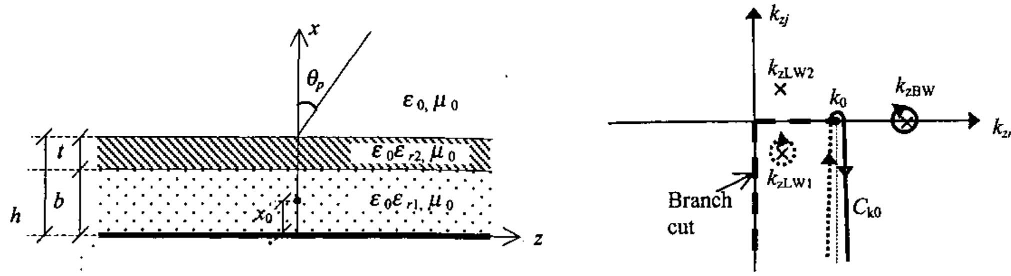

Fig. 1. (a) The substrate-superstrate configuration treated here. *Parameters* used in all the numerical simulations: $\varepsilon_{r1} = 2.1$; $\varepsilon_{r2} = 10.8$. (b) Location of the singularities in the complex k_z plane. k_{zBW}: bound-wave (TE_1) proper real pole; k_{zLW1}: leaky-wave (TE_2) improper complex pole; k_{zLW2}: complex-conjugate of the k_{zLW1} pole; k_0: branch point. The steepest-descent path C_{k0} lies partly on the improper Riemann sheet (dotted line), and partly on the proper Riemann sheet (solid line).

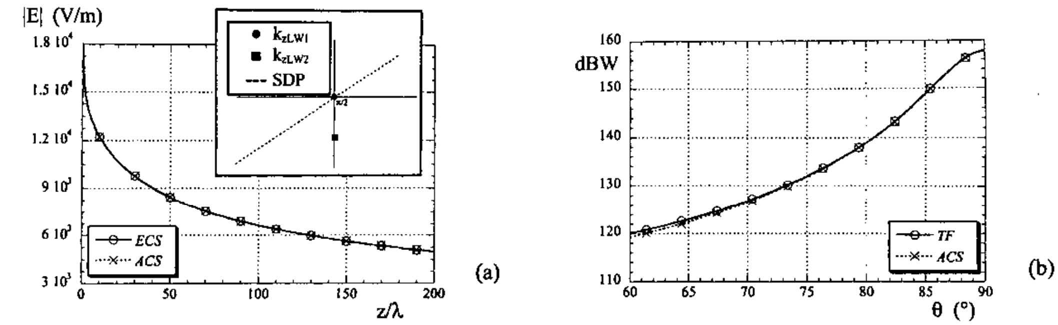

Fig. 2. Comparisons for (a) the near and (b) the far field, at $f = 19.5$ GHz. The location of the poles is shown in the insert figure in the steepest-descent plane of the variable ϕ defined by $k_z = k_0 \sin(\phi)$, where the steepest-descent path (SDP) through the point $(\pi/2, 0)$ is also shown. *Legend*: ECS: exact continuous spectrum; ACS: approximate continuous spectrum; TF: total field.

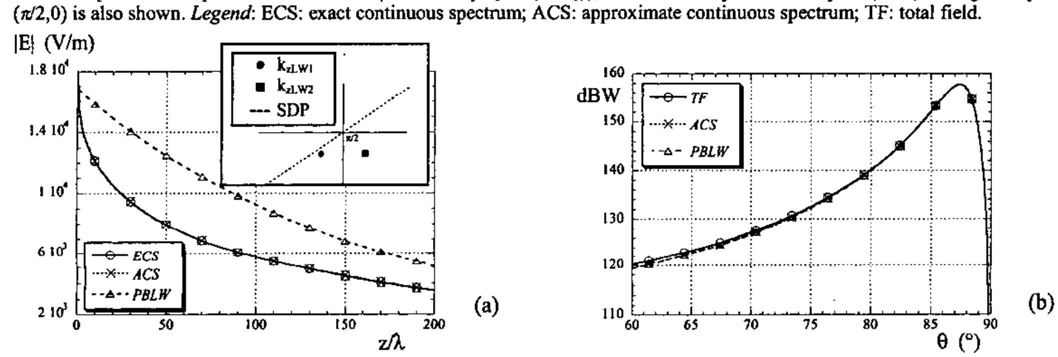

Fig. 3. Same as in Fig. 2, at $f = 19.4844$ GHz. *Legend*: PBLW: power-based leaky wave.

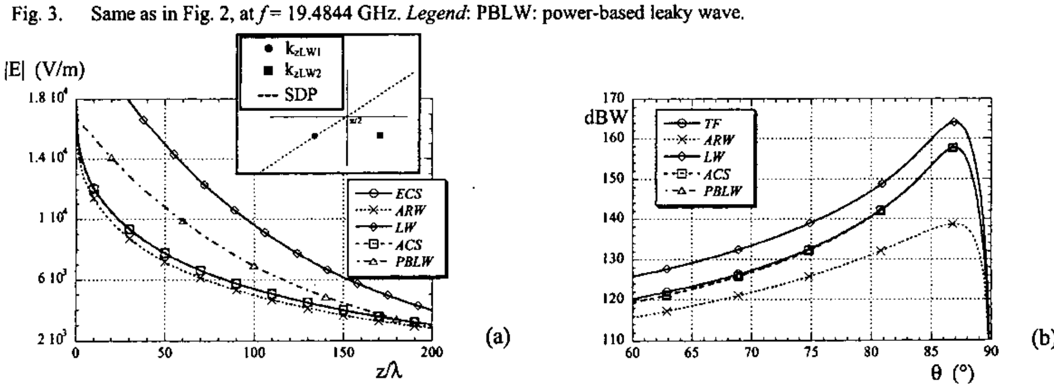

Fig. 4. Same as in Fig. 2, at $f = 19.4781$ GHz. *Legend*: LW: leaky wave; ARW: approximate residual wave.